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Making the Right (Discourse) Moves: Facilitating Discussions in the Mathematics Classroom

The mathematics classroom envisioned by the NCTM *Principles and Standards* (2000) is one in which teachers deliver fewer monologues and engage in more dialogues with students. The teacher is not an ordinary participant in mathematics classroom discussion but plays a special role in facilitating and steering discourse. Calls for encouraging discourse in mathematics classrooms are pervasive, and the analysis of discourse has become a prominent theme in current mathematics education research. Nevertheless, while many teachers may feel the goal is a worthy one, some may also feel at a loss as to the specific strategies or techniques that may be used to encourage and facilitate meaningful mathematical discourse among their students.

This department consists of articles that bring research insights and findings to an audience of teachers and other mathematics educators. Articles must make explicit connections between research and teaching practice. Our conception of research is a broad one; it includes research on student learning, on teacher thinking, on language in the mathematics classroom, on policy and practice in mathematics education, on technology in the classroom, on international comparative work, and more. The articles in this department focus on important ideas and include vivid writing that makes research findings come to life for teachers. Our goal is to publish articles that are appropriate for reflection discussions at department meetings or any other gathering of high school mathematics teachers. For further information, contact the editors.

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This article has three goals, all designed to meet the needs of teachers who wish to enrich mathematical discourse in their classrooms. First, we describe a framework in which teachers can have "discourse about discourse," that is, a structure for teachers to organize their thinking about how their moves can influence mathematical discussions. We then provide specific examples of techniques used to facilitate mathematical discourse, couched in terms of this framework. Finally, we address some of the needs and concerns of teachers as they use mathematical discourse to shift their classroom learning environment to a more participatory model.

THE FRAMEWORK—ANYONE CAN DANCE

Heaton (2000) likens her experience as a teacher participating in classroom mathematical discourse to an improvisational dance performance. Indeed, the subtitle of her book is *Relearning the Dance.* Far too often, however, mathematics teachers feel as though they are stuck in rush hour traffic, gridlocked with topics to cover and no room to maneuver, much less dance. So we will start with simple moves anyone can make, such as practicing "wait time."

Thirty years ago, the notion of allowing "wait time" following a teacher's question to a student or group of students was brought to the attention of both researchers and practitioners (Rowe 1974). The key finding was that the time many teachers allowed between the posing of a question and their explanation of its answer was simply too short to provide most students with any reasonable opportunity to consider the question, much less to formulate

a possible response. Out of this realization came a recommendation that was as remarkably simple as it was effective in changing discourse in the classroom: Teachers could consciously wait 3 to 5 seconds (literally counting to themselves "one thousand one, one thousand two, . . . ") following a question to actually allow students time to think about it. Teachers adopting the technique of forcing themselves to allow this "wait time" found their questions moving from being merely rhetorical signposts in a monologue to real invitations for students to engage in a dialogue. We see wait time as a discourse move, that is, a deliberate move a teacher makes in order to manage or influence discourse in the classroom.

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Heaton's metaphor is that mathematical discourse in the classroom is a dance. Krussel, Edwards, and Springer (2004) extend that metaphor, recognizing that the teacher not only is a participant but also serves special managerial and directorial roles. These multiple responsibilities are similar to those of the lead member of an improvisational dance troupe. These responsibilities cast the teacher in multiple roles:

- *Choreographer:* The teacher plans the (mathematical) storyline for the class period.
- *Stage manager:* The teacher sets the stage for classroom discourse, providing the backdrop (mathematical context) and the props (tools, manipulatives, other resources).
- *Director:* The teacher structures the scene (classroom setting) and determines the rules and roles of the players (students). For example, the teacher may arrange students in small groups engaged in collaborative problem solving and assign students specific responsibilities during the process. After some time, the teacher may move the students into a whole class setting for the purpose of sharing strategies and results.
- *Dancer:* The teacher participates in the improvisational performance (the discourse). Improvisational dancing (discourse) requires careful observation and reaction to the moves of the other dancers (participants), and one's responses should create further opportunities for the other participants to continue.

In order to play all these roles effectively, teachers need a broad repertoire of moves as well as a deep understanding of the form and function of each move. Just as a dance move is deliberate and influences the next moves in the dance, a discourse move is a deliberate action taken by a teacher to encourage, facilitate, participate in, or influence the discourse in the mathematics classroom. Krussel, Edwards, and Springer (2004) describe a teacher's discourse move according to these essential characteristics:

- *Purpose:* the intended aim of the teacher in making the move
- *Setting:* any time constraints, student groupings, tools available, prior history
- *Form:* verbal—challenge, probe, invitation, or hint; nonverbal—facial expressions, gestures, etc.
- *Consequences:* both immediate and long-term cognitive and attitudinal effects the move has on students

Finally, as director and stage manager, the teacher needs to decide when to improvise and when to stick to the script. The decisions a teacher makes about what to say or do next during a class period may follow a carefully laid out plan. A formal presentation or exposition of material by the teacher might follow such a plan to the letter, but a classroom in which students are expected to engage actively in discourse with one another and the teacher will rarely play out exactly as envisioned. For this reason, it makes sense to distinguish teacher discourse moves by their spontaneity. Krussel, Edwards, and Springer (2004) acknowledge these distinctions by making the following classification of discourse moves:

- *Scripted moves* are those planned in advance and carried out in a predetermined sequence. For example, the moves the teacher makes in setting the context or task and the structure for discourse are often scripted.
- *Provisional moves* are planned in advance but made conditionally on anticipated "cues" or "benchmarks." For example, the teacher may plan to move the class from small group work to a full class discussion, but only when satisfied that all groups have reached certain conclusions or results.
- *Improvisational moves* are made "on the fly" in response to unanticipated developments in the discourse. For example, an unexpected strategy employed by a student or small group of students in the solution of a task requires quick evaluation by the teacher of its mathematical integrity and a decision about an appropriate response. If the strategy has merit, the teacher must also decide whether there is value in sharing the strategy for wider discussion, and if so, how. Making appropriate improvisational moves is the most demand-

ing challenge in the discourse-rich classroom, and adequate content knowledge and pedagogical content knowledge are critically important for making appropriate improvisational moves.

Now let's examine wait time as a simple discourse move and see how the framework illuminates its characteristics. According to the framework, wait time as a discourse move has the following essential characteristics:

- *Purpose:* to encourage students to address a question as meaningful instead of regarding that question as rhetorical or as simply part of a teacher's monologue
- *Form:* simple and nonverbal; a pause
- *Setting:* again, quite simple: it follows any question
- *Consequence:* Students are faced with an expectant silence, forcing them actually to address the question instead of waiting for the teacher to answer it.

Wait time casts the teacher as director, cuing the students to take roles as participants in the discourse. Further, wait time as a move is scripted at the global level; that is, the intention on the part of the teacher is to try to make this move a habit of mind following most, if not all, questions. Explicitly acknowledging the purpose, form, setting, and consequences of the wait time move, as well as characterizing the move as a habitual, scripted move used by a teacher who is purposefully shifting his or her role toward director of a participatory discourse, clarifies what the move is and why one would want to consider using it. In a nutshell, this is the purpose of the discourse framework: to give teachers a practical way of looking at discourse moves and a common vocabulary for discussing their shapes and uses. Next, we will learn to dance a little better.

EXPANDING THE REPERTOIRE OF MOVES

In reviewing the research literature on mathematical discourse, the unit of discourse analysis under study can vary widely. For example, the analysis may focus on the significance of particular utterances made by a student or teacher. In other cases, entire sequences or patterns of exchanges between classroom participants are studied. The analysis may be at an even broader level, perhaps examining the establishment of a classroom norm or climate for discourse that has evolved over several weeks or months. Similarly, dance moves may be simple and short or complex and heavily choreographed. In this section, we will add a few more moves to our repertoire while retaining the moves that we believe anyone can make with practice.

Allowing wait time is a particularly simple example of a discourse move (and decidedly nonverbal). *Revoicing* is a discourse move that is as simple to understand as wait time but allows greater variation in form and varies in consequence, depending on its setting. As a result, the effect of this move on classroom discourse can be targeted more closely. Recent research refers to *revoicing* as a move made when one person repeats, summarizes, rephrases, translates, or recasts the contribution of another participant in a discourse (Cazden 2001; Forman and Ansell 2002a, 2002b; O'Connor and Michaels 1996). This definition admittedly allows for considerable complexity in the nuances the teacher may bring to this move. In fact, revoicing exactly what someone else has just spoken is rather difficult. Emphasis is placed on different words; the tone and the timbre of the voice change; concern, incredulity, or surprise may be introduced in the revoicing.

Given that each discourse move has purpose, form, setting, and consequence, what is the purpose of revoicing a student's vocal contribution? In a typical question-response-evaluation discourse pattern, the teacher steers the class through the reconstruction of a mathematical or logical process (solving an equation, creating a proof, etc.) by posing questions at each step, allowing a student to contribute a response, and then vocalizing an evaluation of that response. The pattern is repeated until the process is complete. Invariably, it is the teacher who evaluates the responses, while the students watch for opportunities to respond with some hope for approval and then await judgment of their contributions. When the teacher chooses instead to revoice a student's contribution, he or she in effect passes the baton to the students to evaluate the response. The form of the revoicing may give the students clues for evaluating the response, or the teacher may append a query to give the students direction in evaluating the response. Either way, one consequence is that an important discourse norm undergoes change: The teacher is including, if not deferring to, student voices in evaluating student contributions. Another common consequence of revoicing is to move evaluation one step away from the contributor and one step closer to the contribution. Without abrogating the contribution as his or her own, the teacher who revoices focuses student attention on the words spoken and away from the speaker. Together, these two

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consequences work to encourage students to take the initiative in evaluating each others' contributions to a mathematical discourse, shifting classroom discourse norms in the process. We now have the following profile for revoicing as a discourse move:

- *Purpose:* focus class or group attention on the vocal contribution of a student
- *Form:* echoes that of the student's contribution
- *Setting:* varies, e.g., serves as alternative to question-response-evaluation pattern in a group or full-class setting
- *Consequence:* varies from legitimizing the contribution to allowing an error, but in general fosters autonomy and encourages student evaluation of the contributions of peers

Now consider the role a teacher assumes when revoicing: She or he becomes director and actor simultaneously. As director, the teacher draws the attention of the class to the student's contribution and invites reflection and comment; but the teacher also signals to the class that their continued participation is valued and encouraged and that in effect he or she wishes to be a fellow actor. Additionally, revoicing spans a variety of discourse, from fully scripted, to provisional, to improvisational. This flexibility makes it a useful move to always keep in mind.

Compared to the wait time move, revoicing requires a bit more attention from the teacher. The wait time move invariably increases the quality of student contribution; revoicing, on the other hand, must be done selectively. In addition, the form of the move must be considered in order to maximize the desired consequences. For example, recent research efforts (Forman and Ansell 2002a, 2002b; O'Connor and Michaels 1996) have observed a wide variety of consequences based on the revoicing move. Revoicing can introduce mathematical terms in the discourse; focus on alignments and oppositions of arguments; legitimize student contributions; signal opportunities for students to agree or disagree; level the playing field; allow an error; and foster autonomy.

Clearly, these consequences depend in large part on the form and setting of the move, which differentiates revoicing substantially from wait time. Although more complex than wait time, revoicing benefits from clarification through the framework as a move a teacher makes when transitioning from director to actor. This technique fulfills a variety of needs loosely focused on increasing student reflection on the contributions of other students.

Finally, we turn to the technique of modeling in classroom discourse those habits of mind that the mathematics teacher wishes the students to internalize. These habits are more about process than algorithm and represent values of the mathematical community. In fact, they are often mentioned when the discussion turns to thinking mathematically. We focus on two examples: collective reflection and problem solving.

Cobb et al. (1997) use the term *collective reflection* to refer to a type of discourse concerned with considering an action or process as an object of reflection. Reflective abstraction is a Piagetian notion used to explain similar (internal) activity by an individual. Cobb et al. (1997) are quick to point out that participation by an individual student in the collective reflection of a class is a condition of the habit of reflective abstraction, but collective reflection does not by itself ensure that the student will develop this habit. Those who see value in a participatory model of learning, however, place a certain emphasis on developing the ways in which the learner participates in the activities of a community. Communal reflection on accepted and challenged processes is one of the important activities of any community.

Problem solving is another type of mathematical discourse commonly undertaken by the community of mathematicians. George Pólya (1957), for example, is well-known for his enumeration of problem-solving strategies. How can a teacher's discourse moves explicitly focus student attention on the process of problem solving and set the stage for students to internalize them as proper tools for use in mathematical discourse?

For a class initially faced with solving a problem, the teacher discourse move of inquiring whether a particular strategy (working backward, simplifying the problem, etc.) would be fruitful is not so much an invitation to collective reflection as it is an implicit recognition that the vocalized strategy is a valid one in the mathematical community of practice. Such a move stands one level higher in both scope and complexity than the previous example of revoicing: It assumes the teacher has at hand a ready and explicit knowledge of these strategies and can tailor the discourse move to match current student contributions with an effective strategy. However, asking the class to characterize the strategy or heuristic process that the class has just used to solve a problem communally via discourse is an invitation to reflect collectively. It is also a challenge to each individual to engage in the metacognitive act of reflecting about his or her own thinking.

Clearly, the setting, form, and purpose of collective reflection and problem solving contribute more equally to their consequences than they do to the consequence of wait time and revoicing moves. The purposes of scaffolding the choice of process to one of a number of best practices in the community as well as of moving the class to collective reflection on a process just undertaken makes these moves both more complex and more valuable. For one

thing, this type of move, though itself scripted, is an invitation for contributions from the class that will entail subsequent teacher moves that are at least provisional if not improvisational. Also, this kind of move blurs the roles of teacher as director and as fellow dancer in Heaton's metaphor. As with the examples of wait time and revoicing, collective reflection and problem-solving strategies lead the classroom discourse toward a more participatory model of learning, which requires and acknowledges the contributions of students to their own learning in that special community of practice represented by the mathematics classroom.

FROM INTERPERSONAL TO INTRAPERSONAL COMMUNICATION: DANCING WITH YOURSELF

Teachers of mathematics are under growing pressure to increase student participation in classroom discussions, and yet they have been given very few practical ideas on how to accomplish this feat. The discourse framework presented here is an ongoing effort to provide teachers and researchers a lens for viewing and a vocabulary for discussing discourse moves. The classification of moves in terms of teacher role (via the dance metaphor) and level of spontaneity provides a way to clarify not only what the move is but when and why to use it. In essence, the purpose of the framework is to move the classroom to a more participatory model of learning, in which the teacher takes on a role at once more central and at times less visible: directing, participating in, and improvising within a mathematical discourse with students. As a parting note, consider what Sfard (2000, 2001) calls "thinking as communicating." The intent is to consider thinking as a type of communication; that is, thinking itself is intrapersonal communication (talking to yourself, perhaps silently, even nonverbally). A participatory model based on thinking as communication acknowledges the importance of modeling in interpersonal communications those activities and processes (collective reflection, problem solving, etc.) that the community values as ways of thinking mathematically. All the moves described here have as their ultimate goal rich and fruitful mathematical discourse in the classroom. Let's dance!

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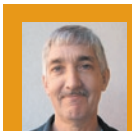

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