Getting Students to Mastery

Mathematical Practices for Deep Understanding
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A steady dose of formative assessment is the key to getting students comfortable with the mathematical practices the Common Core standards require.

Ask a random sample of adults (excluding any math teachers) what they dislike most about math, and you'll probably hear two words: word problems. But when we ask adults this question in the future, their responses might be radically different—if their teachers helped them get accustomed to using the Common Core Initiative's recommended standards for mathematical practice while they were in school. Maybe they'll even answer, "I loved word problems!"

These eight mathematical practices are the antidote to teaching mathematics as a series of "plug and chug" procedures ("Do this for this kind of problem"). Students who make sense of word problems don't start by asking, "What kind of problem is this?" They start by trying to figure out what the problem means. What does the problem ask? What information is given, what needs to be found, and which mathematical procedures will lead to that information? Students will approach word problems with questions like these once they master the mathematical practices. They'll not only be able to solve mathematically rich problems, but they'll also appreciate math's usefulness.

Students won't master the standards for mathematical practice overnight, even after schools begin implementing the Common Core State Standards. The practices address habits of mind, thinking processes, and dispositions that help students develop "deep, flexible, and enduring understanding of mathematics" (Briars, Mills, & Mitchell, 2011, p. 20). Teachers will need to both give students problems that require them to use the practices and create environments that support student discourse and risk taking. And students will need a steady diet of feedback on their performance. Math educators will need to change three things to nurture these practices. Besides changing instructional strategies and materials, teachers will need to change their assessments (so items measure mathematical practices as well as computational skills), and make their feedback focus on students' mathematical reasoning, modeling, and other practices—not just on correct answers. Instruction, assessment, and feedback will all need to focus more on higher-order thinking skills, communication, and collaboration.

The Key: Formative Assessment

Formative assessment can be a powerful tool to integrate cognitively demanding tasks into students' math work. Teachers should share with students clear learning targets that include the eight practices. Because a "target" isn't a target unless students are aiming for it, the first step is finding ways to show students what this kind of mathematical thinking looks like and how they'll know whether they are engaging in it. Students must have clear "look-fors" to monitor the quality of their work and their thinking. Most important, lessons should include ungraded opportunities for students to try out strategies, construct mathematical arguments, and receive feedback on their efforts.

Helping Students Think About Their Thinking

To explore how a teacher might leverage the power of formative assessment, let's look at how 3rd grade teacher Renee Parker modified her instruction to help students develop Mathematical Practices 1 (make sense of problems) and 3 (construct viable arguments). Renee and her colleagues developed a student-friendly rubric to improve their learners' ability to communicate about their thinking while solving problems. They devised five problems, each of which required students to draw diagrams, write number sentences, and explain their reasoning and the steps they used to solve that problem. They then focused on one problem each week (Parker & Breyfogle, 2011). The rubric provided a clear set of things students should look for in their work (for example, "Was all the important information from the problem used?"). The students discussed weekly problem work with their teacher and classmates, so they got a clearer idea of items on the rubric (for example, "I write what I did and why I did it") looked
like. Working through these problems helped all Parker's students learn, but her lower-achieving students improved the most. Students especially improved in writing explanations that included mathematical vocabulary and strategies. This project included changes to all three areas—instruction, assessment, and feedback. Instruction came to include clear targets, which were clarified by the rubric, student work examples, and discussion around both. Assessment changed: The weekly problems required students to demonstrate that they could use mathematical tools, mathematical models, and viable arguments. These assignments were formative; they allowed students to practice and improve, think intentionally about their own reasoning, and talk with others as they continued to reflect and learn. Feedback changed: It included dialogue about rubric items that helped students make sense of the problems.

**Teaching with the Mathematical Practices in View**

Zeroing in on one problem shows how a teacher might change these three elements of lessons to help learners simultaneously master both that problem and the mathematical practices. The sample word problem in Figure 1 is aimed at 5th graders and maps to the domain of problem solving at Depth of Knowledge Level 3 on the Common Core framework. The problem asks learners to first compute the total number of crayons in four boxes holding 64 crayons each, and then find the answer to several other questions focused on how to divide these crayons among 32 students. Math problems at this level "require reasoning, planning, or use of evidence to solve problems … citing evidence and developing logical arguments for concepts" (Webb, 2002, p. 10).

**Figure 1. Sample 5th Grade Word Problem**

Mrs. Brown bought 4 boxes of crayons at the store to share with her students. Each box contained a total of 64 crayons.

**Part A**
What is the total number of crayons Mrs. Brown bought at the store? Explain your answer using diagrams, pictures, mathematical expressions, and/or words.

**Part B**
Mrs. Brown wants to give each of her students an equal number of the crayons she bought. There are 32 students in Mrs. Brown's class. How many crayons should each student get?

**Part C**
How many more boxes of crayons does Mrs. Phelps need if she wants each of her students to get 12 crayons? Explain your answer using diagrams, pictures, mathematical expressions, and/or words.

We use this problem because it's a good example of the kind of work students need to do to become skilled with the mathematical practices. Let's look at how teachers might shift their teaching methods to reinforce the practice of constructing viable arguments as students tackle this problem.

**Instruction.**

Instructional objectives for this lesson should include something about students explaining their reasoning (not just, "Students will solve problems using two-digit multiplication and addition"). You might give students learning targets like, "I can explain what I did and why I did it" or "I used mathematics language."

After sharing the learning targets, show students examples of mathematical explanations former students made for a sample problem. As a group, discuss how one might rate these various explanations, using a rubric that measures how well a student constructs a viable mathematical argument. Rubric items might include "use definitions of mathematics words correctly" or "use drawings to explain steps in solving the problem."

Give students a few minutes to read the problem in Figure 1 and think about how they would approach Part A. Ask each learner to share with a partner what they think this part of the problem is asking and their plan for solving it.
Remind students to ask questions like "Why are you choosing to solve it that way?" as they listen to their partner's explanation.

As students solve Part A individually, circulate around the room, noting different approaches students use. Have a few kids explain their solutions to the whole class. Encourage students to refer to the mathematical practices rubric as they listen to these explanations and to question any presenter whose explanation isn't clear. Next, have students respond in their math journals to a prompt like, "Reflect on how you explained 'what you did and why you did it' in solving this problem."

Ask students to solve the rest of the problem—Parts b and C—in pairs and then share their solution with another pair. To wrap up, students might write about what they learned about constructing viable arguments.

Assessment.

Student work should give you lots of evidence about their skills at constructing or explaining mathematical arguments—and what they should do to improve. Evidence comes from three sources: watching students work, talking with students about their process, and observing the final product. Give ongoing, formative feedback, both oral and written, that will move students forward as they take their next steps. Most important, structure classes so that students have immediate opportunities to take those next steps.

Feedback.

Feedback can foster improvement in mathematical practices in four ways:

- Feedback should center on the practice. If students are to develop skill at writing an explanation of how they solved the sample problem, your feedback should be about their explanation. For example, if a student wrote for Part A, "I subtracted 4 from 64 and got 60," asking a student "Why did you subtract?" would be more effective than crossing out "subtract" and writing "multiply." A student who multiplies because you told him to won't necessarily learn how to explain when one should multiply.

- Feedback should be descriptive. Good feedback comments on what the student did and makes at least one suggestion for a next step. For example, if a student wrote $64 \times 4 = 256$ for an explanation in Part A, responding, "You've identified the right mathematical expression. Can you tell me why that's the right mathematical expression?" would be better than "Great!" or "Write more."

- Feedback takes into account how students learn mathematics (Daro, Mosher, & Corcoran, 2011)—and differs depending on where students are in their learning trajectory with the content. For example, if a student explains his or her work in Part A by writing $64 + 64 + 64 + 64 = 256$ or drawing pictures to express the same reasoning, you might point out that the student is correct and ask whether he or she can think of any explanations that involve a process other than addition. Such feedback affirms her current understanding and helps push his or her thinking; as students develop an understanding of multiplication, they generally grasp additive reasoning before multiplicative reasoning.

- Feedback helps students see that there are more and less effective arguments, as well as correct and incorrect ones. Imagine one student wrote for Part C: "Students have 8 crayons from 4 boxes. To get 12, each student needs 4 more crayons, that's half as many as they already have. So half as many more boxes is 2. 4 boxes + 2 boxes is 6 boxes." Say another student wrote: "12 crayons \times 32 students = 384, so we need 384 crayons. 384 \div 64 in a box = 6, so we need 6 boxes." As feedback to each student, you could say, "You've both written clear explanations that get you to the same number of total boxes. Work together to compare your explanations. Are they both right? If so, how is that possible? Explain to one another the next step you'll take to answer the question."

Applying These Principles

We've zeroed in on one of the eight mathematical practices and one word problem, but the principles illustrated in this example apply to helping students develop skills in all eight practices and in a wide range of mathematical content.
Note that the changes to instruction, assessment, and feedback we’ve portrayed here all involve formative assessment strategies: communicating clear learning targets and criteria for success, designing performances of understanding that match the learning targets, providing feedback that feeds students forward, and asking questions that make student thinking visible. Teachers will need a thorough understanding of what each practice looks like and how students typically learn mathematics content to use these strategies effectively.

All these strategies are interdependent. Focusing instruction on the quality of students’ mathematical arguments won’t work unless you’ve communicated to students what a mathematical argument is—and made constructing one their learning target. Having a target won’t help unless students have multiple opportunities to solve word problems in various ways and talk with others about that problem solving. The formative learning process is a cycle. At its best, it’s an upward spiral, getting students to mastery.

### Cognitively Demanding Math Tasks

To strengthen the eight Standards for Mathematical Practice, assign tasks that ask students to

- Use accurate vocabulary and appropriate symbols and labeling to explain their thinking and justify their solutions.
- Represent problems in multiple ways and reason with models and pictorial representations.
- Use technology tools to demonstrate a solution to a problem, and justify their tool selection.
- See and use structure to solve problems, notice patterns in calculations, and look for ways to generalize what they learn to broader mathematical strategies.

### References


### Endnote

1 The eight mathematical practices are make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. For a detailed description of each practice, see [www.corestandards.org/Math/Practice](http://www.corestandards.org/Math/Practice).
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