Roosevelt School District: June 6th, 2014

Mathematical Practices that Promote Rigor & Complexity in Student Thinking

RSD June 6th, T. Lauffer, Facilitator
Learner Benefit:

Why does this particular topic have relevance for you in your current role and work?

RSD June 6th, T. Lauffer, Facilitator
(Enthusiastically read aloud together 😊)

Today, as learners, we will:

1) ... be able to define and describe rigor & complexity in lesson design, instruction, and student thinking in observable classroom terms.

2) ... be able to articulate and support the value of productive struggle in the learning/thinking process.

3) ... be able to identify teacher & student actions that demonstrate the 8 Mathematical Practices.

4) ... practice interactive and metacognitive skills, confronting our externalized and internalized dispositions towards learning mathematics.

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Norms for our Learning Today

1) Start on time after breaks and lunch.
2) Be actively engaged and listen to others.
3) Follow the signal to close out talking.
4) Say: “I don’t know” to own your learning.
5) Feel the stretch of productive struggle.

1) Come back late. No one will miss you.
2) Passively observe or dominate the thinking.
3) Hold side or electronic conversations.
4) Hide what you need or want to learn.
5) Feel frustrated and let that shut you down.

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Mathematics Metacognition Journal

**Noticing my thinking:**

**Summary Statements:**

**Observations to Explore:**

**Questions to Answer:**

**Noticing my feelings:**

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Formative Assessment of Prior Knowledge & Paradigms

1) Explain how these three terms describe the math instructional shifts: focus, coherence, rigor?

2) If the math standards are more focused= fewer, how can they be more rigorous?

3) What is the difference and significance of conceptual understanding vs. procedural understanding?

4) What is the relationship between the math standards and the 8 math practices?

5) What is the role of productive struggle in the thinking/learning process? How might this push up against existing paradigms regarding learning math?
Cross-referencing the LOI tool with the rigor & complexity of the AZCCRS

We need to see teachers designing for rigorous & complex learning.
We need to see teachers instructing with rigorous & complex learning activities and tasks.
We need to see teachers assessing and evaluating student thinking and work against a higher level standard.

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Sharing a common definition of rigor.
(Source: Rigor is not a Four-Letter Word, Barbara Blackburn)

Creating an environment in which
➢ each student is expected to learn at high levels,
➢ each student is supported so that he/she can learn at high levels, and
➢ each student demonstrates learning at high levels.

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A concrete demonstration of how rigor and complexity work to produce thinking

What do you notice in this demonstration?

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What you should see in teacher practice to support rigorous and complex student thinking:

<table>
<thead>
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<tbody>
<tr>
<td>Teacher planning to CREATE student involvement &amp; cognitive engagement thru a learning design that requires</td>
<td></td>
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<tr>
<td>➢ Engagement</td>
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<td></td>
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<tr>
<td>➢ Accountability</td>
<td></td>
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<tr>
<td>➢ Making thinking visible.</td>
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</table>
Describe the difference between students doing the work and students thinking?

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Can you see people thinking?
What can you see?

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Foundational Understandings about “Thinking”  
(Source: Making Thinking Visible, How to Promote Engagement, Understanding, and Independence for All Learners)

When asking someone to think, what are we asking him or her to do mentally?

How would we see that in a classroom observation in teacher or student action?
Draw/Describe a picture that represents “cognitive engagement”

1) CONNECTION TO ME OR MY INTERESTS
2) ACADEMIC RIGOR = PUSHES ME
3) LIVELY TEACHING = REQUIRES MY INVOLVEMENT
Let’s look at our opening task:
What did the design require of individuals?
What did the design tell me as the facilitator?

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Constructing thinking starts with two things:

**Context:**
When am I going to use this or need to know this?

**Purpose:**
Why do I need to know this? Of what value is this to me?
Ah ha! (chicken or the egg)

Understanding is the outcome of thinking.
It is through the process (or learning task) of
✓ Applying
✓ Analyzing
✓ Evaluating
✓ Creating
that we gain understanding.
So what’s the common problem here?
## “Levels of Thinking”

<table>
<thead>
<tr>
<th>BRAIN FUNCTION</th>
<th>ASSESSMENT</th>
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<tbody>
<tr>
<td>Knowledge:</td>
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<tr>
<td>Comprehension:</td>
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<tr>
<td>Application:</td>
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<td>Analysis:</td>
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<td>Evaluation:</td>
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<tr>
<td>Synthesis:</td>
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### Lower Level Thinking

- Application of Concepts
- Generalizations
- Thought Processes

### Higher Level Thinking

**TRANSFER of**
1. Concepts
2. Generalizations
3. Thought Processes

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“Levels of Thinking”

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<tr>
<td>Knowledge:</td>
<td>Same form I gave it to you: Repeating the same back</td>
<td>Repetition, memory strategies: write/say it</td>
</tr>
<tr>
<td>Store and recall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comprehension:</td>
<td>Different form: identify examples/non-examples. Restate it</td>
<td>Using examples to help the learner make connections to which he can relate</td>
</tr>
<tr>
<td>Understand (connect to prior knowledge)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Application:</td>
<td>New situation: Solve a problem/ do a task</td>
<td>Examine the thinking process steps. Meta-cognition. Ask questions</td>
</tr>
<tr>
<td>Use knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis:</td>
<td>New situation: dissect look for relationships, report what observed</td>
<td>What is the criteria you will look for as evidence, what thought process to use</td>
</tr>
<tr>
<td>Take knowledge a part</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evaluation:</td>
<td>New situation: use criteria, draw a conclusion and cite evidence</td>
<td>What criteria to use, evidence to cite, what thought process to use</td>
</tr>
<tr>
<td>Make a judgment/conclusion based on criteria/evidence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synthesis:</td>
<td>Invent something new</td>
<td>A thought process for creativity</td>
</tr>
<tr>
<td>Creating something</td>
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Examples of Learning Engagement by Design in Mathematics

(More effective design) | (Less effective design)

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8 Kinds of “Thinking Moves” integral to creating understanding

1. Observing closely & describing what’s there
2. Building explanations & interpretations
3. Reasoning with evidence
4. Making connections
5. Considering different viewpoints & perspectives
6. Capturing the heart & forming conclusions
   (identifying the essence of something)
7. Wondering and asking questions
8. Uncovering complexity and going below the surface

(Source: Making Thinking Visible: How to Promote Engagement, Understanding and Independence for all Learners)

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What kinds of “thinking routines” can teachers use to help student gain metacognition skills?

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<td>Teacher planning &amp; implementation to MAINTAIN student involvement during learning work.</td>
<td></td>
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<td><em>What extra supports might which students need?</em></td>
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What is Scaffolding?

Scaffolding puts one in position to do a job.
And what else?
Examples of Scaffolding
Examples of Scaffolding

- Asking guiding questions
- Chunking information, breaking down processes
- Differentiating by content, process, product
- Color-coding steps of a project
- Using wait time as thinking time
- Using visuals and graphic organizers to map thinking
- Providing tools like interactive reading guides
- Modeling and guided practice with feedback
- Providing clear expectations (what makes a good answer)
- Presenting multiple opportunities to learn

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<td>Teacher actions to SUSTAIN student involvement when learning feels like a struggle.</td>
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<td>➢ Teacher Questions</td>
<td></td>
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<tr>
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<td>➢ Student Responses</td>
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<td>1. Format Matters</td>
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<td></td>
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<td>2. Higher-Order Thinking (HOT) questions asked by teacher</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Open-ended questions asked by teacher</td>
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Format Matters:

Teacher uses question stems to complete.

Teacher requires the use of academic vocabulary and accurate math terminology in speech.

Students respond in a complete sentence using the stem as a starter.

Students speak in an audible VOICE to be heard and clearly understood.

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What’s the difference between H.O.T. vs. Open-Ended Thinking Qs in math?

Higher Order Thinking Qs:

Open-ended Thinking Qs:
In the opening exercise, which were comprehension, application, analysis or evaluation questions?

LOOK at the QUESTIONS
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<td>Responses by students and how teacher pushes the thinking</td>
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<td></td>
<td></td>
<td>1. No opt out</td>
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<td></td>
<td>2. Right is right</td>
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<tr>
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<td></td>
<td>3. Stretch it</td>
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“No OptOut response to students (doesn’t accept “I don’t know” as work avoidance)

1) Teacher provides correct answer.
2) Another student provides the answer.
3) Teacher provides a cue and has the student try again.
4) Another student provides a cue.

1) Student repeats answer.
2) Initial student repeats the answer.
3) The student uses the cue to find the answer by breaking it down.
4) The initial student uses the cue to answer.

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Cues: additional useful information to help in the thinking process

Some examples of cues are as follows:

✓ The place where the answer can be found...
✓ The step in the process that’s required at the moment...
✓ Another name for the term that’s a problem...
“Right is right” response to students
(partially to all the way right—hold to a high standard of correctness)

- Ask to extend the answer; hold out for all the way. “Tell me more. Think more deeply.”
- Repeat back the answer & vocally question its incompleteness
- Use wait time as thinking time to probe.

- Answer the question; note that the student answered correctly but to a different question.
- Repeat the question to cue a sharper focus for what is being asked.
- Use technical or academic vocabulary.

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“Stretch It” response to students
(extends knowledge & check for understanding)

- Ask “How or Why?”
- Ask for another way to answer
- Ask for a better word
- Ask students to explain their thinking, which is different than show your work
- Ask for evidence
- Ask students to integrate a related skill
- Ask students to apply a skill in a new way
- Ask for comparisons, opposites, or non-examples

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LUNCH TIME!

A MATH DISCUSSION-STARTER:

Which number doesn't belong?

9  16  25  43

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“Mathematical Practices” Defined:

✓ These describe *varieties of expertise* that teachers should seek *to develop in students.*

✓ These practices rest on important "*processes and proficiencies*" with longstanding importance in mathematics education.

✓ These Mathematical Practices are listed throughout the grade level documents to reflect the need *to connect the mathematical practices to mathematical content* in instruction.
8 Mathematical Practices

1) Make sense of Problems & Persevere in Solving Them
2) Reason Abstractly & Quantitatively
3) Construct Viable Arguments & Critique the Reasoning of Others
4) Model with Mathematics
5) Use Appropriate Tools Strategically
6) Attend to Precision
7) Look for & Make Use of Structure
8) Look for & Express Regularity in Repeated Reasoning.

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4 Thinking Tasks to learn the Math Practices (MPs) in application

1. **Discuss** and digest the 8 MPs
2. **Examine** the MPs in the context of the Math Standards progression K-5
3. **Recognize and identify** how **daily student behaviors** in learning mathematics leads to gaining proficiencies in the MPs
4. **Analyze and align** **student mathematical proficiencies** with the various MPs.

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(Math Practice 1) Make Sense of Problems and Persevere in Solving Them

*Thinking TASK #1 (Table Group Discussion of all 8 Practices)*

*Thinking TASK #2 (Examining the Relationship of the 8 MP to the Math Standards)*

Mathematically Proficient Students Can:

*Thinking TASK #4 (Matching Descriptors to the Wall Posters of Each Math Practice)*

1) 
2) 
3) 
4) 
5) 

<table>
<thead>
<tr>
<th>Classroom Routines/Structures:</th>
<th>Teacher Practices/Questions:</th>
<th>Student Behaviors Observed:</th>
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<tr>
<td></td>
<td></td>
<td><em>Thinking TASK #3 (Matching Student Behaviors to each Math Practice and having a line-up check)</em></td>
</tr>
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1\textsuperscript{st} Table Thinking Task: Digesting the Math Practices (key ideas)

- Select a table facilitator and a timekeeper.
- Assign each table member a math practice.
- Engage in table dialogue around 2 questions:
  1) What does each math practice mean?
  2) What is its value for thinking?
- Use your journal as a “thinking template” at pause points.
- We will check in with you in half way through.
2nd Table Thinking Task:
Recognizing the math practices in relationship to the Math Standards progression at each grade level

Arizona Mathematics Standards Articulated by Grade Level

<table>
<thead>
<tr>
<th>Operations and Algebraic Thinking</th>
<th>Standards</th>
<th>Mathematical Practices</th>
<th>Explanations and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</td>
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<tr>
<td>Students are expected to:</td>
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| K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problems. This applies wherever drawings are mentioned in the Standards.) Connections: K.OA.2; K.W.2; K.SL.2; ET00-S1C4-01; ET00-S2C1-01 | K.MP.1. Make sense of problems and persevere in solving them. K.MP.2. Reason abstractly and quantitatively. K.MP.4. Model with mathematics. K.MP.5. Use appropriate tools strategically. | Using addition and subtraction in a word problem context allows students to develop their understanding of what it means to add and subtract. Students should use objects, fingers, mental images, drawing, sounds, acting out situations and verbal explanations in order to develop the concepts of addition and subtraction. Then, they should be introduced to writing expressions and equations using appropriate terminology and symbols which include “+”, “−”, and “=”.
- Addition terminology: add, join, put together, plus, combine, total
- Subtraction terminology: minus, take away, separate, difference, compare Students may use document cameras or interactive whiteboards to represent the concept of addition or subtraction. This gives them the opportunity to communicate their thinking. |
2nd Table Thinking Task:
Recognizing the math practices in relationship to the Math Standards progression at each grade level

Examine the progression grades K-5 of this standard from the domain “Operations & Algebraic Thinking.”

Notice and note on the margins of your poster:
✓ The progression of the rigor and complexity
✓ The demonstration of the math practices with the standard
✓ Anything else you see as significant. You will notate and exchange your thinking/observations with your table partner A or B.
3rd Table Thinking Task:
Matching student behaviors that demonstrate the math practices

✓ Each table has a poster with the 8 Math practices noted on it.
✓ Open your envelope and find particular student behaviors that could be observed in the classroom which are a part of the process of gaining that mathematical proficiency. *(This is what you would see students doing.)*
✓ Match the grouping of student behaviors to the math practice it best demonstrates.

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4th Table Thinking Task:
Describing the Math Practices by student proficiencies

✓ Each table group will be given 4 different descriptors of competencies completing this stem: “Mathematically proficient students can…….”
✓ Read through the proficiency descriptor carefully & discuss which math practice it best demonstrates in student actions. (things a student can do)
✓ Match the descriptor strip to the math practice posted using the tape on your table.
✓ Raise your hands when your table is done.
Let’s talk about Productive Struggle

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“Productive Struggle”

What is its value in the thinking process?

What makes it productive?

How do students, teachers, parents perceive learning struggle?

How do we reshape perspectives on productive struggle to meet the challenge of rigor?
What does this cartoon say about current attitudes?

- WHAT ARE YOU DOING? HOMEWORK??
- I WASN'T SURE I UNDERSTOOD THIS CHAPTER, SO I REVIEWED MY NOTES FROM THE LAST CHAPTER AND NOW I'M REREADING THIS.
- YOU DO ALL THAT WORK??
- WELL, NOW I UNDERSTAND IT.
- HUH! I USED TO THINK YOU WERE SMART.
What does this quote say about failure.

“I have not failed. I've just found 10,000 ways that won't work.”

Thomas A. Edison
Change the message, change the mindset.

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How to set up struggle which is productive for learning

1. Make clear the content, context, and value of the learning (the what, why, and when)

2. *Know your learners so that you can scaffold appropriately to push & expand their zone*

3. Practice “metacognition” and thinking skills in routines, modeling, and language

4. *Reinforce the “growth mindset” that focuses on effort and process – not instant knowing.*

5. Tell students/parents overtly that rigor forces stretching and your belief that their child can do it, and you will be there to help him succeed!
What am I currently seeing in classrooms, and how do I give feedback to change that?
(Enthusiastically read aloud together 😊)

Today, as learners, we will:

1) ... be able to define and describe rigor & complexity in lesson design, instruction, and student thinking in observable classroom terms.
2) ... be able to articulate and support the value of productive struggle in the learning/thinking process.
3) ... be able to identify teacher & student actions that demonstrate the 8 Mathematical Practices.
4) ... practice interactive and metacognitive skills, confronting our externalized and internalized dispositions towards learning mathematics.
Learner Benefit:

Why does this particular topic have relevance for you in your current role and work?

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In summary, this is the **BEST** you can give your students

B elief in their ability to learn
E ncouragement to persevere
S upport as they are on the path
T ime to exert effort and try again
Sources Used in Designing this Session:

- *Rigor is not a Four-Letter Word*, Barbara Blackburn
- *Making Thinking Visible: How to Promote Engagement, Understanding, and Independence for All Learners*, Ron Ritchhart, Mark Church, Karen Morrison.
- *Teach Like a Champion: 49 Techniques That Put Students on the Path to College*, Doug Lemov
- *Thinking Through Quality Questioning: Deepening Student Engagement*, Jackie Acree Walsh and Beth Dankert Sattes