Task Analysis/Lesson Plan Outline for Constructing the Meaning of Pi

7.G.B.4. Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Because of the relationship between radius, diameter, and circumference:
7.RP.A.2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.

Common Errors / Misconceptions:
Students may believe that pi is an exact number instead of understanding that 3.14 is just an approximation of pi. Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

Focus

7.G.B.4. Know the formulas for the ~~area and~~ circumference of a circle and solve problems.

Lesson 1	• Use a real life situation to learn the names of the parts of a circle (center, diameter, and radius; circumference, arc, and chord will be later). Record terms on drawings in journals. • Write an equation (d = 2r or r= ½ d). Given the length of the diameter or the radius, find the radius or the diameter respectively. • Construct circles with a compass when given a specific length for radius or the diameter. (Keep constructed circles for lesson 2.) • Journal – Draw diagrams and write about understanding	**Rigor**: *CU*- radius is half of a diameter; diameter is twice the radius; APP-Situation to introduce circles **Coherence**: fractions and decimals; writing equations
Lesson 2	• Use lesson 1 context to learn the term circumference. • Measure the diameter and radius of several circular items. (Use both metric and standard. Use both fractions and decimals.) Examine the data measurements (diameter and circumference of the a variety of circular items) to determine a relationship. • Notice a relationship between the lengths of the diameter and the circumference, e.g., as the length of the diameter increases the circumference increases. • Students compare additively and relatively to arrive at the conclusion that the circumference is 3 and some more times the diameter and are introduced to pi. • Use measuring tools to develop a conceptual understanding of pi as a ratio of the circumference and the diameter. • Estimate the length of the circumference or diameter of a circle given the diameter or circumference respectively. • Journal -Use diagrams and sentences that explain the meaning of pi.	**Rigor**: *CU, A* **Coherence**: **Fluency:** Estimating lengths using understanding of pi
Lesson 3	• Complete the following statements: - The circumference is _____ the diameter. - The diameter is about _____ of the circumference. - Pi is _____. - For every one unit of measurement of the diameter, there are _____in the circumference. • Use the statements above to write a formula for the circumference of a circle and be able to explain why the formula makes sense. • Work several problems in context related to finding the lengths of the radius, diameter or the circumference, given one of the dimensions. • Work problems with no context related to finding the lengths of the radius, diameter or the circumference. • Journal: Stacy says she does not understand what pi means. In your journal, describe how you would explain pi to her?	**Rigor**: *CU, App, FL* **Coherence**: unit rate; writing and solving equations

Lesson 1

Daily Math Framework

Math Review (10 minutes)	
Skill review: (not new material) • Share 3-5 problems a day with students • Students solve problems in their notebooks or math journals. • Five minutes of work time and five minutes to correct. • Five minutes students share the various ways they solved the problem.	• Give students a maximum of 2 word problems. • Display problems electronically. Differentiate by giving students different number sets, when appropriate • Set timer for both work time and sharing time. Note: Teaching as students are working, indicates that either the kind of problems or the numbers chosen need to be reconsidered. Observations may result in the identification of a small group of students who many need to be pulled for reteaching and extra practice work on this concept.
Mental Math (5 minutes)	
• Ask for benchmark measurements they know • White boards for displaying answer • Show pen, about how long do you think this pen is? If it is 5 inches, how long is this (letter opener)? • Same: pen 5 “, pencil? • Pencil: 7 ½ in , letter opener? • Same: purple pen 6 in, show length of half of pen using fingers… • Same: pencil 19cm, use half in a week, now length? • If use 2/3 about how long is pencil?	Purpose of this activity to have S - reason about measurement(in & cm) without measuring - use their imagination and apply fraction concepts mentally - plant ideas that standard and metric measurements can be used (in preparation of the lesson) Note: using tasks connected to lesson because measurement is not a strength for most Ss
Concept Lesson (30-40 minutes) **Instructional Approach: Construct Knowledge**	
Standard: Investigate the relationships of the attributes of a circle. **Substandard**: Know the parts of a circle. Jaime’s dog, Rade, loves to be outside but sometimes he digs holes to get out of the fenced yard. Jaime decided to tie him to a short pole dug deep in the yard so the 10’ rope would not wrap around the pole. Rade does not like the rope so when he walks, he walks as far away from the pole as he can. • Imagine his walk. • Draw a diagram to show Rade’s path. • Put an X in any spot that Rade might be standing and show the rope. • Share your diagram with your partner and discuss any measurements you know. • Have students present (briefly). • Make sure to ask, *Why did Rade’s path form a circle?* • That length that is always the same (the rope) is called the **radius**. Write, say and Ss repeat. • Ask them to draw another radius on their diagram. *What do you know about that radius?* (10’) *Can you draw more? How many more?* *When you have more than one radius, we say radii.* Write, say and repeat. • Point to what you think is the **center** of the circle. Write, say and Ss repeat.	At this time do not share the specific math terms or relationships. As Ss experience the concepts, terms will be introduced and then practiced. Walk around, observe and listen to the students to determine which pair will present. Important ideas to pull out through questioning: • The distance from the pole to Rade is 10’. • The distance from the pole to Rade will always be 10’ no matter where he is if he is always as far away from the pole as possible. • Rade’s path is a circle because he was always the same distance away from the pole. • The pole is at the **center** of the circle.

Jaime's sister has a dog, Rade2. The two dogs do not like each other so they are always as far apart from each other as they can be. • Show both Rade and Rade 2 on your circle. • *How do you know that they are as far apart as possible?* • Share your diagram with your partner and discuss any measurements you know. • *Why does it make sense that the distance is 20'?* • Have students share. • Introduce the term **diameter**. Write, say and Ss repeat. • *Can your circle have more than one diameter? How many more?* (infinite number) • Assign groups to write a definition for either radius or diameter. *The only thing I will tell you is that the radius of Rade's circle will always be 10' and the diameter of the circle will always be 20'.* • Have groups present and negotiate until they can reach consensus about the definition. • Have students use technology or textbooks to compare their definitions to the textbooks definitions. Discuss any differences. Have students use words to describe the relationship between the radius and the diameter. Use a stem if necessary. The diameter is_____ the radius. The radius is ___ the diameter. *Use your word description to write an equation that shows the relationship between a radius and a diameter of the same circle. Use d and r.* • Practice: finding radius, given diameter; finding diameter, given the radius (do not limit to whole numbers) Use writeboards to observe understanding. • Practice: using a compass to draw a circle with a given diameter or a given radius. Measure the diameter and the radius to verify correct drawings.	Important ideas to pull out through questioning: • The distance from Rade to Rade 2 is 20'. I know this because the distance from the center to each dog is 10'. We are not looking for a textbook definition of a circle but one that shows understanding of the attributes of these parts. Definition of radius: a segment from the center of a circle to the circle; distance between 2 points (1 is center, 1 is on the circle). Definition of diameters: a segment that goes through the center of a circle and has endpoints on the circle; a segment formed from 2 radii (if this comes up ask Ss if they can draw 2 radii that do not form a diameter. Since they can this definition does not have enough specificity. The doubling and halving concept of radius and diameter is not difficult. This practice focuses on student use of the language and of additional practice with fractions and decimals. Students need to learn to use a compass and recognize that the consistent radius forms a circle. This will help them when later they have to define a circle as the set of points equidistant from one point.
Closure (5-10 minutes)	
Journal: Draw a diagram that shows a circle, its center, a radius and a diameter. Write a paragraph that explains what you know about a circle. Have students share their writing with a partner. Have some share with the whole class. Class decides if they agree, disagree, want to add to, etc. *So, today you learned – use the recording to summarize the important concepts of the lesson.*	Record important ideas as students share. Possibly share other ideas you saw or heard from students while walking around.
Sub groups:	