AlphaShapes Soup
by
Peter Fiore

A 5-day lesson plan for Mathematics Grade 8 in which students will explore a wide variety of Geometric concepts utilizing Cuisenaire AlphaShapes manipulatives.
What are AlphaShapes?

AlphaShapes, manufactured by Cuisenaire Company of America, is a collection of 26 interrelated shapes. These translucent plastic shapes, ideal for use on an overhead projector, come in two colors. One set contains 52 pieces (2 sets of 26, one green and one orange): 6 different triangles, 11 different quadrilaterals, 3 different pentagons, 3 different hexagons, 1 circle, 1 oval, and 1 half circle. Each shape is identified by a letter of the alphabet, and each angle of every polygon is numbered.

The set of 26 AlphaShapes of one color have a sufficient variety of properties—number of sides, measures of length and angle, parallelism of sides—for students to explore a wide range of relationships. Used together with an angle ruler (to measure length and angle) and a transparent centimeter grid (to measure area), Alpha Shapes provide multiple experiences in developing definitions and in forming hypotheses and formulas related to measurement of length, angle, perimeter, and area.

Overall Objectives

The students will:

- Informally learn about the properties and relationships among the various AlphaShapes pieces.
- Demonstrate their abilities in contrasting and classifying geometric shapes according to their properties.
- Enhance their capabilities in applying geometric language.
- Assimilate understanding of the meanings of the regions in a Venn Diagram.
- Explore physically particular mathematical definitions, which leads to a deeper understanding of what the mathematical definitions are.
- Discover that different definitions of a shape can be equivalent.
- Explore various techniques for finding area.
- Investigate the relationship between area and perimeter by discovering that shapes of the same area do not necessarily have the same perimeter.
• Discover different area relationships in ways that will later lend more meaning to comparatively abstract formulas.
• Develop physical intuition that will help them make sense of the standard formulas concerning the sum of the angles of a particular shape.
• Measure the radius, diameter, circumference, and area of a variety of circles, enabling them to search for patterns and discover relationships among these measurements.

**NCTM Standards**

• Students should precisely describe, classify, and understand relationships among types of two-dimensional objects using their defining properties.
• Students should understand relationships among the angles, side lengths, perimeters, and areas of similar objects.
• Students should create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
• Students should describe sides, positions, and orientations of shapes under informal transformations such as flips, slides, and scaling.
• Students examine the congruence and similarity of objects using transformations.
• Students should draw geometric objects with specified properties, such as side lengths or angle measures.
• Students should understand both metric and customary systems of measurement.
• Students should understand relationships among units and convert from one unit to another within the same system.
• Students should understand, select, and use units of appropriate size to measure angles, perimeter, and area.
• Students should use common benchmarks to select appropriate methods for estimating measurements.
• Students should select and apply techniques and tools to accurately find length, area, and angle measures to appropriate levels of precision.
• Students should develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles, and develop strategies to find the area of more-complex shapes.
**New York State Standards**

- Students will apply a variety of reasoning strategies, using pictures, diagrams or patterns, and identifying similarities and differences among a wide variety of problem types. (1A)
- Students will make and evaluate conjectures and arguments, using appropriate language, discriminating relevant from irrelevant information, expressing solutions clearly and logically, understanding that there is no one right way to solve mathematical problems, but that different methods have different advantages and disadvantages, and clarifying problems using discussion with peers. (1B)
- Students will make conclusions based on inductive reasoning, applying strategies and results from simpler problems to more complex situations. (1C)
- Students will develop and explore models that do and do not rely on chance, exploring the range of possibility (certainty, impossibility, sometimes). (4G)
- Students will use appropriate tools to construct and verify geometric relationships, using compasses, ruler, and protractors, and identifying the properties of congruent and similar triangles. (4I)
- Students will select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy, understand the uses of units and square units, finding the measures of angles, and determining the degree of accuracy needed in measurement situations. (5B)
- Students will develop measurement skills and informally derive and apply formulas in direct measurement activities. (5C)
- Students will develop critical judgment for the reasonableness of measurement. (5F)
- Students will identify, describe, represent, extend, and create geometric patterns. (7A)
- Students will describe and represent functional relationships using rules and verbal descriptions. (7B)
- Students will demonstrate an understanding of congruence between two geometric figures and what congruence means about the relationships between the sides and angles of the two figures. (7F)
- Students will use the sum of the number of degrees of measure of triangles, quadrilaterals, hexagons, etc. to solve problems. (7G)
• Students will understand and use proper terminology, symbols, definitions, and formulas for undefined and defined terms. (7H)

Resources


Materials & Equipment

• Sets of Alpha Shapes (one set of 26 for every two students).
• Notebook paper and pencils.
• 3 x 5 cards.
• Overhead projector.
• String or yarn.
• Scissors.
• Dictionary or math glossary.
• Transparent centimeter grids (one for each student & teacher).
• Angle rulers (one for each student & teacher).
• Circular objects of different sizes (coins, jar lids, paper cups, paper plates, Frisbees).
Overview

I. Day One: Introduction (45 minutes)
   A) Free Exploration (15 minutes)
      1) Introduction to AlphaShapes
      2) Unstructured exploration
      3) Student reports
      4) Introduction of terminology
   B) Same and Different (15 minutes)
      1) Students work in pairs, each choosing a shape
      2) Students list similarities and differences
      3) Student reports.
   C) Treasure Hunt (15 minutes)
      1) Students given clues, working in pairs
      2) Pair justifies choice/extension of knowledge

II. Day Two: Labels On Loops (45 minutes)
   A) Student activity using Venn Diagrams (25 minutes)
   B) Class discussion (15 minutes)
   C) Extending the knowledge: Homework (5 minutes)

III. Day Three: Properties Of Shapes (45 minutes)
   A) Preparation: 20 Questions (10 minutes)
   B) Student activity (20 minutes)
   C) Class discussion (10 minutes)
   D) Extending the knowledge: Homework (5 minutes)

IV. Day Four: Alternative Area (45 minutes)
   A) Demonstration (5 minutes)
   B) Student activity (20 minutes)
   C) Class discussion/solutions (15 minutes)
   D) Extending the knowledge: Homework (5 minutes)

V. Day Five: Life Is A Circle (45 minutes)
   A) Demonstration of measurement techniques (5 minutes)
   B) Student activity (20 minutes)
   C) Class discussion/solutions (15 minutes)
   D) Extending the knowledge: Homework (5 minutes)
Day One: **Introduction**

**Abstract**
This lesson is designed to introduce the students to AlphaShapes, ignite their abilities to classify geometric shapes by their properties, separate shapes by their differences in properties, and thus strengthen their mathematical vocabularies.

**Objectives**
Upon completion of the lesson, the students will:
- Become familiar with AlphaShapes manipulatives.
- Classify the shapes according to their similarities and differences.
- Utilize reasoning strategies to determine shapes with a minimum of clues.
- Strengthen and amend their mathematical vocabularies.

**Materials**
Classroom materials needed for this lesson include:
- A set of 26 AlphaShapes for every 2 students.
- Notebook paper and pencils.
- 3 x 5 cards.
- An overhead projector.

**The Lesson (45-Minute Class)**

**Free Exploration** (15 minutes)

**Focus & Review**
Remind students that their prior knowledge pertaining to geometric shapes will be pertinent to the lessons presented this week. Ask them to be aware of the key ideas and terminology put forth during the lesson. Divide the class into groups of two and pass out to each pair a set of 26 AlphaShapes.

**Unstructured Exploration**
Allow the students some time (5 minutes) for unstructured exploration with the AlphaShapes. Most students will begin to experiment with the shapes without additional directions, but the following suggestions may be beneficial:
- “Can you cover any shapes completely with other shapes?”
- “How many different shapes can you find?”
- “Find out all you can about the shapes and make a list of your
findings on your notebook paper.”

**Student Reports**

Have the students read an idea from their lists while the rest of the class simultaneously verifies the idea using their AlphaShapes.

**Introduction of Terminology**

Geometric terminology should be introduced as the students share their ideas with the class so they have an opportunity to add to strengthen their mathematical vocabulary. Using geometric terms such as parallelogram, trapezoid, and congruent within the context of responding to the students suggestions creates a deeper connection to the meaning of the words.

**Same and Different (15 minutes)**

*Students work in pairs, each choosing a shape.*

Have students work in pairs, and ask each group to select one AlphaShape. Then give the following directions:

**Students list similarities and differences**

“Make your own lists of all the similarities and differences you can think of for the two pieces. Don’t show your list to your partner. I will time you at exactly 2 minutes. When the time has expired, compare your lists with your partner. You will get one point for each similarity or difference that is not on your partner’s list, and the person with the most points wins the round. A game consists of three rounds. I will wait until everyone is completed with the first round before moving on to the second, so that I can time everyone at once. We will stop at the end of the third round and discuss how the game went and what we learned.”

**Student reports**

After the third round, the students are asked to share their work. The teacher then pinpoints the class discussion toward the various characteristics of the AlphaShapes, allowing the teacher an opportunity to determine the prior knowledge of the students concerning geometric shapes.

**Treasure Hunt (15 minutes)**

*Students given clues, working in pairs*

On the overhead projector, prepare a set of clues for one of the shapes, such as the following for AlphaShape H:
Then give the following instructions:

"I have written on the overhead a set of three clues, which apply to one and only one of the 26 AlphaShapes in front of you. I am going to reveal the clues to you one at a time. Work with your partner to find the shape, and if you think you have it, then raise your hand.

(Vocabulary within the clues may include some terminology which is unfamiliar to the students. In this fashion, they may be forced to make their selection by the process of elimination.)

Pair justifies choice/extension of knowledge

When a pair is sure it has found the shape that justifies all the clues, ask one of the members to hold it up. Ask the group to justify its selection. Pose the following important questions:

"Is it possible that there is more than one AlphaShape that satisfies these clues? Why or why not?"

To extend the students knowledge, assign each pair of students a different AlphaShape, and assign them the homework task of producing on a 3x5 card a set of three clues for their particular shape. Ask them to make the clues increasingly more specific as they move from the first to the second to the third.
Day Two: **Labels On Loops**

**Abstract**
This lesson is designed to develop the students' geometric reasoning skills through the usage of Venn Diagrams, inducing the students to decide in which of the four regions of the Venn Diagram a particular geometric shape belongs, depending upon its classification.

**Objectives**
Upon completion of the lesson, the students will:
- Further develop their understanding of the properties of geometric shapes.
- Increase their awareness of the meanings of the various regions of a Venn Diagram.
- Experience greater proficiency regarding their reasoning skills.
- Gain additional enhancement of their mathematical vocabularies.

**Materials**
Classroom materials needed for this lesson include:
- A set of 26 AlphaShapes for every two students.
- Construction paper and scissors.
- String or yarn.
- Scissors.
- A set of labels for each pair of students.

**The Lesson: Labels On Loops (45-Minute Class)**

**Student Activity using Venn Diagrams (25 minutes)**
Divide the class into pairs, with each pair receiving a set of 26 AlphaShapes. Each pair will also receive two 3-foot pieces of yarn or string. Lastly, they will be given a set of labels identical to the one depicted below:
Then give the following directions:

“Make two overlapping groups of string like this. (Picture on the board.) As a warm-up, let’s pick the label ‘There are three angles’ and place it above the left-hand loop. Now let’s find all the shapes that can be placed in that loop. Then let’s pick the label ‘There are four sides’ and placed it above the right-hand loop. Now let’s find all the AlphaShapes that can be placed in that loop. Place the rest of the shapes outside both loops.”

“With these two labels, are there any shapes which will be placed in the overlapping section of both loops? Why or why not?”

“Now let’s play the game. One person picks two labels, and places them face down on the loops. The other person tries to guess what the labels are by placing a piece somewhere inside. The first person tells the second whether or not the placement was correct. If the first person says ‘Yes,’ leave the piece in place. If the first person says ‘No,’ either move the piece or remove it and try putting down another piece.”

“The object is to guess the labels in as few tries as possible. I will move around the room to see how the games are going. If someone does guess the two labels correctly, then switch roles. We’ll stop with about 20 minutes left and discuss how things went.”

**Class Discussion (15 minutes)**

The following questions can be used to prompt interesting student responses:

“What strategies did you use to select shapes to try in the game?”

“Can anyone think of a label that applies to exactly half of the
AlphaShapes?"

“Which pairs of labels make it possible to fill all four regions of the Venn Diagram? Which pairs make it impossible?”

“Could you design some other shapes different from the AlphaShapes that would fit into a particular region?”

“Are there any other labels besides these which could be used in this activity?”

After noting the ideas of the students, two critical points should be addressed. First, in order to determine what a label says, it is just as important to discover what is outside a loop as what is inside. Second, when pairs of labels are contradictory, the intersection of the loops will certainly be empty.

**Extending the Knowledge/Homework (5 minutes)**

Students are to work with their partners on this assignment. Have them consider only the quadrilaterals among their AlphaShapes. They are to use the labels “All angles are congruent” and “All sides are congruent” with a Venn Diagram. In doing so correctly, they will find that the loop labeled “All angles are congruent” will contain all rectangles, and the loop labeled “All sides are congruent” will contain only rhombuses. The intersection of the two loops will contain squares. This exercise will help the students comprehend that squares are a special kind of rectangle, that all squares are rectangles, and that all squares are also rhombuses.
**Day Three: Properties Of Shapes**

**Abstract**
This lesson is designed to enable students to explore physically particular mathematical definitions using manipulative, leading to deeper understanding of what mathematical definitions are. It does so by showing that different definitions of a shape can be equivalent.

**Objectives**
Upon completion of the lesson, the students will:

- Classify those AlphaShapes that satisfy a given collection of properties.
- Realize that different definitions of a shape can be equivalent—in other words, there can be more than one way to describe the same set of shapes.

**Materials**
Classroom materials needed for this lesson include:

- A set of 26 AlphaShapes for every 2 students.
- Notebook paper and pencils.
- An overhead projector.
- A dictionary or math glossary.
- 3x5 cards (for homework)

**The Lesson (45-Minute Class)**

**Preparation: 20 Questions (10 minutes)**
As a warm-up, one student stands in front of the class with a secret shape. The rest of the students have to determine the shape by asking questions that involve a “Yes/No” response. For example, if the class had to guess AlphaShape S, the answer to the question “Does the shape have 3 sides?” would be No, since S has four sides.

The object of the game is to determine the shape by asking as few questions as possible. This will develop the students' skills at gathering information from various clues, and eventually they will come to a realization that certain questions are unnecessary if their answers can be inferred from answers to previous questions.

Play as many rounds as possible for the full ten minutes.
**Student Activity (20 minutes)**

On the overhead projector place an acetate of the following while giving each pair of students a copy as well:

**Properties Of Shapes**

You will need 26 AlphaShapes of one color for every two students.

1) Below are some properties of a shape. Find all the AlphaShapes that satisfy *all* of these statements.
   a) There are four sides
   b) There are at least two congruent sides.
   c) There are four angles.
   d) The shape has at least one line of symmetry.
   e) The shape has a right angle.
   f) There are two pairs of congruent angles.
   g) All angles are congruent.
   h) There is at least one pair of parallel sides.
   i) There are two pairs of parallel sides.
   j) Both diagonals cut the shape into two congruent parts.

2) Cover up one of the properties above so that the same set of AlphaShapes you found for Part 1 still satisfies the remaining properties. Then cover every other property that can be covered without making the set change. Make a list of the properties that remain.

Uncover all of the properties and repeat this, but try to do it in a different way so that a different list of properties remains. Compare your lists with those that others have made.

3) Take AlphaShapes K, O, Q, U, and W (parallelograms). List all the properties that are shared by *all* these shapes. Make sure that no other AlphaShapes satisfy all the properties you have listed. Then repeat Part 2, using that list.

The teacher should monitor the progress of the activity for 20 minutes.
**Class Discussion (10 minutes)**

Prompt the discussion with the following questions:

“What strategies did you use to find all AlphaShapes that satisfy all of the properties?”

“Why didn’t a particular shape satisfy all the properties?”

“Would your answers be the same if, in addition to AlphaShapes, you could use any shapes that you can draw?”

With respect to the students’ solutions, one approach to Part 1 that should be noted is to actually remove shapes that don’t satisfy each property. There are 11 shapes that satisfy a), but only 8 of the 11 satisfy b), and only 6 satisfy a), b), c), and d). Step e) eliminates all shapes except Q and W, which also satisfy g) through j).

There are many solutions to Part 2: properties a) and g); properties a), e), and i); properties a), e), and j). Since any polygon has the same number of sides as it has angles, properties a) and c) are interchangeable, making the following solutions viable: c) and g); c), e), and i); c), e), and j). Therefore, we have the scenario that there can be equivalent ways to describe the same set of shapes.

The shapes given in Part 3, as stated, are parallelograms. As a set, they satisfy all of the properties in Part 1 except d), e), and g), with the simplest list containing just properties a) and i).

**Extending The Knowledge: Homework (5 minutes)**

Repeat Part 3 of the class activity using a rhombus as the category of shape. On 3x5 cards, have the students write down the properties that apply. Have them determine, in a flow chart approach, whether the existence of one property implies the existence of another by drawing arrows between the cards on the larger sheet of paper. Check the responses the following day, and affix the correct ones to the front board.
Abstract
This lesson is designed to allow students to discover various area relationships that are often taught primarily as formulas. The activity herein encourages students to reason about area relationships in ways that will later give deeper meaning to the formulas.

Objectives
Upon completion of the lesson, the students will
• Explore various techniques for finding area.
• Investigate the relationship between area and perimeter by discovering that shapes with the same area do not necessarily have the same perimeter

Materials
Classroom materials needed for this lesson include:
• A set of 26 AlphaShapes for every 2 students.
• A transparent centimeter grid for every student.
• An angle ruler for each student.
• An overhead projector.

The Lesson (45-Minute Class)
Demonstration (5 minutes)
(Before the students begin the activity, the teacher should demonstrate the following methods for finding area, using the AlphaShapes and a transparent centimeter grid on an overhead projector.)

Counting Squares Using The Grid
Put the transparent centimeter grid under W and count squares. If appropriate, use the formulas for area of a rectangle, showing that it is a shortcut for counting squares (multiplying the lengths of two sides results in 36 cm²). Remind the students that when they use the grid on some shapes, they may need to either count fractional parts of squares or to estimate parts.
Comparing Shapes
Place 2 V's on the overhead projector to show that they form a shape identical to W. Explain that because the two copies of V fit together to form W, V has half the area of W, or 18 cm$^2$.

Combining Shapes
On the grid, put V over C so that only a rectangle is uncovered. The area of this rectangle will be found to be 12 cm$^2$; therefore the area of C is 12 + 18 = 30 cm$^2$. 
**Student Activity (20 minutes)**

On the overhead projector place an acetate of the following while giving each pair of students a copy as well:

**Ways To Find Area**

You will need 26 AlphaShapes of one color, a transparent centimeter grid, and an angle ruler.

1) Compare the area of each piece to the areas of W and Q. Sort the shapes into five piles, as shown:
   a) Area less than W
   b) Area same as W
   c) Area greater than W but less than Q
   d) Area same as Q
   e) Area greater than Q

2) Write about your results and explain how you compared areas.

3) Find the area in square centimeters of at least two shapes from each of the five piles. Estimate you you cannot calculate an exact number. Write about how you found the areas.

4) Find some AlphaShapes with the same area. Measure the perimeters. Do you notice anything?

**Class Discussion/Solutions (15 minutes)**

Prompt the discussion with the following questions:

“How did you decide what strategy to use to find the area of each shape?”

“Which area measurements are rough estimates? Which ones are exact? Why?”

“Did you find shapes which have the same area and different perimeters?”

“Did you find shapes that have the same perimeter and different areas?”

“Did you find shapes that are not congruent that have the same area and the same perimeter?”
Solutions

The areas of 14 AlphaShapes can be found exactly:

Square $W$ has an area of $36 \text{ cm}^2$, found either by counting or by using the formula $A = \text{length} \times \text{width}$.

Rectangle $Q$ has an area of $48 \text{ cm}^2$, by the same methods.

Triangle $V$, one-half of $W$, has an area of $18 \text{ cm}^2$.

Triangle $B$ also has an area of $18 \text{ cm}^2$. Because it can be cut into two triangles such that if copies of these triangles are joined to Triangle $B$, the combination forms Square $W$.

Triangles $I$ and $R$ both have areas of $24 \text{ cm}^2$, as both are half of Rectangle $Q$.

Parallelograms $K$ and $O$ both have areas of $48 \text{ cm}^2$, as each be made from two copies of Triangles $R$ and $I$, respectively.

Triangle $H$ has an area of $24 \text{ cm}^2$, as it is half of Parallelogram
**Hexagon D** has the same area as Square W, 36 cm$^2$. One can make W by cutting a triangle out of one side of the square and pasting it back on the other.

**Trapezoid C**, as explained in Combining Shapes, has an area of 30 cm$^2$.

**Kites G and T** can be found by comparing them to Triangle R. Kite G is comprised of a triangle with a base of 6 and a height of 2 added on to R, while Kite T has the same triangle cut from R. This triangle has an area of 6 cm$^2$. Thus Kite G has an area of 30 cm$^2$, while Kite T has an area of 18 cm$^2$. 
Pentagon M can be seen as Square W with two triangles cut out, both of which have a base of 6, a height of 2, and therefore an area of $6 \, \text{cm}^2$. Thus, M has an area of $36 - 12 = 24 \, \text{cm}^2$.

Extending The Knowledge: Homework (5 minutes)

Working in pairs and using the same materials as in class, try to estimate as precisely as possible the areas of the remaining shapes:

1) Triangle N
2) Rhombus U
3) Trapezoid S
4) Pentagon J
5) Hexagon P
6) Hexagon L
7) Circle X
8) Semicircle Z
9) Ellipse F
10) Quadrilateral A
11) Quadrilateral E
12) Pentagon F

Homework Solutions

1) Triangle N has a base of 6 and a height slightly more than 5, so its area is about $15 \, \text{cm}^2$.

2) Rhombus U is twice N, so its area is about $30 \, \text{cm}^2$.

3) Trapezoid S is made of 3 N's, so its area is about $45 \, \text{cm}^2$. 
4) **Pentagon J** is made from Triangle N and Square W, and thus has an area of 51 cm\(^2\).

5) **Hexagon P** is made up of 6 small triangles, each with an area of N. P therefore has an area of about 23 cm\(^2\).

6) **Hexagon L** can be made from Triangle N by cutting off three small triangles, each of which has an area less than 1. So L has an area of about \(15 - 3 = 12\) cm\(^2\).

7) **Circle X** has an area of about 28 cm\(^2\), by counting.
8) *Semicircle Z* has an area of about 56 cm\(^2\), by counting.
9) *Ellipse Y* has an area of about 37 cm\(^2\), by counting.
10) *Quadrilateral A* has an area of about 20 cm\(^2\), by counting.
11) *Quadrilateral E* has an area of about 17 cm\(^2\), by counting.
12) *Pentagon F*  has an area of about 60 cm\(^2\), by counting.
Day Five: Life Is A Circle

Abstract
This lesson is designed to build intuition about the units being used via direct involvement with measurement, and it brings to life relationships that are sometimes presented as abstract formulas. The dimensions of the circle and semicircle in the AlphaShape set allow concrete investigation of the effects of doubling the diameter on the other measurements.

Objectives
Upon completion of the lesson, the students will:
- Have measured the radius, diameter, circumference, and area of a variety of circles.
- Searched and discovered patterns and relationships among these measurements.

Materials
Classroom materials needed for this lesson include:
- Five circles of different sizes (coins, jar lids, paper cuos, paper plates, Frisbees) for every two students.
- 1 of AlphaShape X and 2 of AlphaShape Z.
- String.
- An angle ruler for each student.
- A compass for each student.
- A transparent centimeter grid for each student.
- An overhead projector.

The Lesson (45-Minute Class)
Demonstration of Measurement Techniques (5 minutes)
Using the differently sized circles noted above, show the students two alternative ways to measure the circumference of a physical model of a circle:
1) By stretching a string around it and then putting the string next to a ruler.
2) By marking a point on the circumference of the circle, and then rolling the circle along a ruler, starting with the marked point touching “0,” and continuing until the marked point again touches the ruler.
**Student Activity (20 minutes)**

On the overhead projector place an acetate of the following while giving each pair of students a copy as well:

**Life Is A Circle**

You will need 5 circles of different sizes (coins, jar lids, paper cups, paper plates, Frisbees), 1 of AlphaShape X, 2 of AlphaShape Z, string, an angle ruler, a compass, and a transparent centimeter grid.

1) Arrange all your circles, including the one made from the two Z’s, in size order.
   a) Estimate in centimeters, then measure the radius, diameter, and circumference of each circle. Record your findings.
   b) Look for patterns and make a list of what you notice.

2) Draw or find a circle whose diameter is half as big as circle X. Use this circle along with circle X and the circle made from the two Z’s to find the following information. Record your answers.
   a) How do the measures of the radii compare?
   b) How do the areas compare?
   c) How does the measure of each circles radius compare to its area?

3) Choose any of your circles. Make two new circles—one whose radius is twice as big and one whose radius is half as big as the circle you choose.
   a) Predict the area of each circle.
   b) Measure the area of each circle.
   c) Explain, in writing, why your findings make sense.

**Class Discussion/Solutions (15 minutes)**

Prompt the discussion with the following questions:
“What patterns did you notice among your measurements?”
“What would happen if you graphed your results?”
“What would happen if you doubled the diameter of the circle? The radius? The circumference? The area?”
“If you were to measure a new circle with a radius of 5, what would you predict to be the measurements of its diameter, circumference, and area?”
“Would your patterns also work for AlphaShape Y (ellipse)?”
Solutions

The students should have discovered the following relationships:

1) The radius is half of the diameter, or the diameter is twice the radius.

2) The circumference is approximately three times the diameter, and about six times the radius.

3) If the diameter is doubled, the radius and the circumference are doubled, but the area is multiplied by 4.  (See if any students have determined that, if the diameter is enlarged by any factor, the radius and circumference are also multiplied by the factor, but the area is multiplied by the factored squared.)

4) The area of a circle is about three times its radius squared.

Extending The Knowledge/Homework (5 minutes)

The students can further investigate why the circumference is roughly three times the diameter of a circle with the following activity:

- Trace Circle X on a sheet of paper, and carefully trace Hexagon P inside of it. Measure the diagonal of P, which is identical to the diameter of X. Measure the perimeter of P.
- Carefully draw another hexagon inside which the circle just fits. Measure its perimeter.
- (The students should discover that the measure of the diagonal of P/the diameter of X is 6 cm. The perimeter of P, therefore, is 18 cm, and the circumference of X, which surrounds P, is slightly more than 18 cm. If accurately drawn, the diagonal of the hexagon surrounding X will be about 7 cm, and its perimeter will be about 21 cm, which is 3.5 times the diameter of the circle. The desired realization by the students is that the circumference of the circle, which lies somewhere between those of the two hexagons, falls between 3 and 3.5 times its diameter.)